Example 3.3.3.

Consider the pursuit-evasion game of two players, whose dynamics are given by (3.1.1) and (3.1.2), with the parameters, $T = 50, \ a = 0.15, \ v_E = 6, \ v_p = 2.5$ and the initial conditions, $y_1^0 = 200$ and $y_2^0 = 120$. Since $y_2^0 > 0$, it belongs to Case 2 stated in Theorem 3.2.2. By solving (3.2.3) with $(y_1, y_2)$ replaced by $(y_1^0, y_2^0)$ and calculating (3.2.12), we have $\delta^* = 15.4002 < T$ and $f_2(120) = 1812400 \geq 0$. Hence the situation considered in this example belongs to Case 2b. If both players execute their strategies optimally throughout the game, we have

$$u^*(t) = \begin{cases} -0.15, & t \in [0, 15.4002], \\ 0, & t \in (15.4002, 50]. \end{cases}$$

$$x_1^*(50) = (6/0.15) \sin(0.15 \times 15.4002) + 6(50 - 15.4002) \cos(0.15 \times 15.4002)$$

$$= -110.31.$$  \hfill (3.3.11)

$$x_2^*(50) = -(6/0.15)[1 - \cos(0.15 \times 15.4002)] - 6(50 - 15.4002) \sin(0.15 \times 15.4002)$$

$$= -220.36,$$  \hfill (3.3.12)

$$w^*(t) = \arg\left[(-110.31 - 200, -220.36 - 120)\right]$$

$$= -2.31, \ t \in [0, 50].$$

$$v = \sqrt{(-110.31 - 200)^2 + (-220.36 - 120)^2 - (2.5)(50)}$$

$$= 112614.45.$$  \hfill (3.3.13)

Also from (3.2.10) with $y_1(T), \ y_2(T)$ and $w$ replaced by $y_1^*(T), \ y_2^*(T)$ and $w^*(t)$, respectively, we have
\[ y_1'(50) = 200 + (2.5)(50)\cos(-2.31) = 115.78, \quad (3.3.14) \]
\[ y_2'(50) = 120 + (2.5)(50)\sin(-2.31) = 27.63. \quad (3.3.15) \]

All the values calculated from (3.3.11)-(3.3.15) are the predictions of final states and the game value for this case if both players execute their optimal strategies throughout the game. The results are summarized in the first row in Table 3.3.3.

Figure 3.3.7 shows the simulation when both players play optimally throughout the game. The computer simulation results are summarized in the second row of Table 3.3.3. It is obvious that the results are consistent with our predictions in the first row of the same table.

Next, with the same parameters and initial conditions, let the evader play optimally and the pursuer play randomly throughout the game. Figure 3.3.8 shows a typical computer simulation results, which are also summarized in the third row of Table 3.3.3. Notice that the resulting payoff is more favorable to the evader than the value calculated in (3.3.13) as expected.

Conversely, with the same parameters and initial conditions, let the pursuer play optimally and the evader play randomly throughout the game. Figure 3.3.9 shows a typical computer simulation results, which are also summarized in the last row of Table 3.3.3. Notice that the resulting payoff is more favorable to the pursuer than the value calculated in (3.3.13) as expected.
Table 3.3.3 Summary of the simulation results for Example 3.3.3.

<table>
<thead>
<tr>
<th>Description</th>
<th>Final states of the players</th>
<th>Game value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expected results</td>
<td>Both players play optimally.</td>
<td>(-110.31, -220.36)</td>
</tr>
<tr>
<td>Fig. 3.3.7</td>
<td>Both players play optimally.</td>
<td>(-110.31, -220.36)</td>
</tr>
<tr>
<td>Fig. 3.3.8</td>
<td>E plays optimally. P plays randomly.</td>
<td>(-132.93, -195.83)</td>
</tr>
<tr>
<td>Fig. 3.3.9</td>
<td>P plays optimally. E plays randomly.</td>
<td>(185.19, -197.96)</td>
</tr>
</tbody>
</table>

Figure 3.3.7 A simulation for case 2b when both players play optimally.